The Mathematical Constant e

The number e is of eminent importance in mathematics, alongside 0, 1, π and i. All five of these numbers play important and recurring roles across mathematics. Like π, e is an irrational number. The value e can be defined a number of ways. We will use the following definition:

$$e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$$

n	e
1	
5	
10	
100	
1000	
etc.	

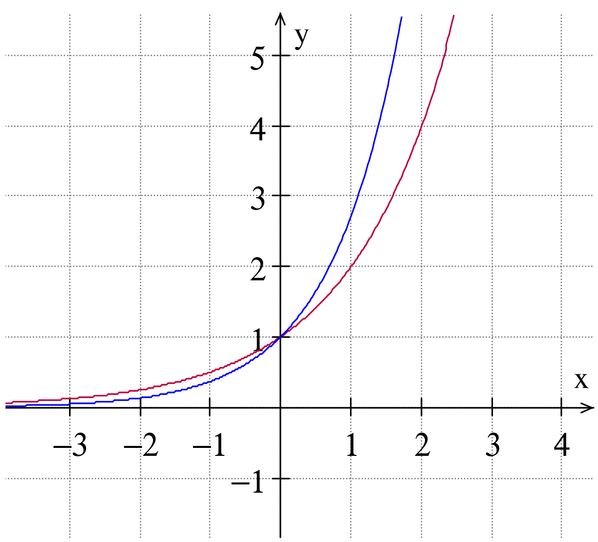
By substituting increasing values of n, we can see that the value of e appears to be approximately equal to __________.

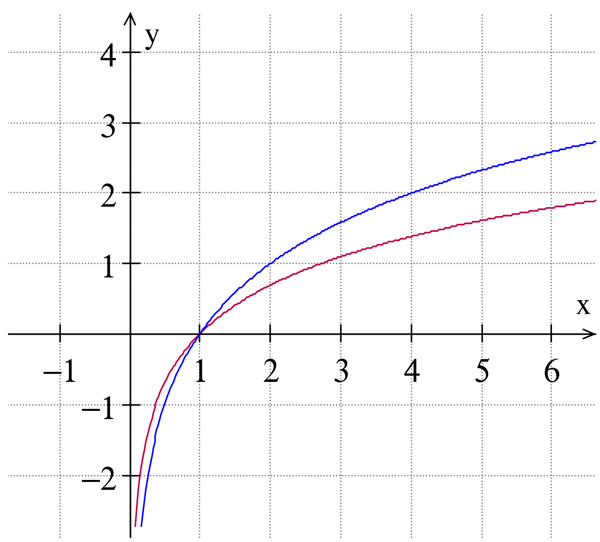
Using a calculator, we can see that the value of e evaluated to 5 decimal places is ____________________.

- Logarithms with base e are referred to as "natural logarithms" and we write $\ln x$ which means $\log_e x$.
- Note that the laws of logarithms also apply to natural logarithms.

Example 1: Compare the Graphs of $y = e^x$ to $y = 2^x$ and $y = \ln x$ to $y = \log_2 x$

Label each function below with the appropriate equation.





Example 2: Identify Characteristics of Graphs of Natural Logarithmic Functions

Identify the following characteristics of the graph of each function:

i. the equation of the asymptote
ii. the domain and range
iii. the y-intercept (to one decimal place)
iv. the x-intercept (to one decimal place)

a. $y = \ln(x - 5) - 4$
b. $y = \ln(-(x - 3)) + 1$

Solution:

a. $y = \ln(x - 5) - 4$

i) Visualize the transformations of the graph of $y = \ln x$:

The graph of $y = \ln x$ has been ____.

Therefore the equation of the ____________________ asymptote is ________.

ii) Domain: ____________________ Range: ____________________

iii) y-intercept: ________

iv) x-intercept: ________

b. $y = \ln(-(x - 3)) + 1$

i) Visualize the transformations of the graph of $y = \ln x$:

The graph of $y = \ln x$ has been ____.

Therefore the equation of the ____________________ asymptote is ________.

ii) Domain: ____________________ Range: ____________________

iii) y-intercept: ________

iv) x-intercept: ________

Example 3: Evaluate Expressions Containing Natural Logarithms

Evaluate each expression.

a. $4\ln e + 5\ln 1 - \ln e^3$ b. $e^{\ln 12 - 3\ln 2}$

Solution:

a. $4\ln e + 5\ln 1 - \ln e^3$

b. $e^{\ln 12 - 3\ln 2}$

Example 4: Solve Natural Logarithmic Equations

Solve the following equations.

a. $\ln x = 2\ln 4 + \ln 3$ b. $3\ln 2x + 4 = 10$ c. $6 + 5e^{2x} = 21$ d. $\log_2 e^{-4x} = 5$

Solution:

$\ln x = 2\ln 4 + \ln 3$	$3\ln 2x + 4 = 10$	$6 + 5e^{2x} = 21$	$\log_2 e^{-4x} = 5$

Example 5: Solve a Problem Using Natural Logarithms

The temperature, T, in degrees Celsius, of a cup of hot chocolate *t* minutes after it is made is given by the equation $T(t) = 92e^{-0.06t}$.

a. Calculate the temperature of the hot chocolate 8 minutes after it is poured.

b. How long will it take the hot chocolate to cool to 50°C?

Solution:

a. Substitute t = 8 into the equation and solve for T.

b. Substitute T = 50 into the equation and solve for t.